Team Contest

1. Three rogues, each with two bags, want to cross a river. They have a boat with three seats, each of which can be occupied by a person or a bag. However, neither of the rogues is willing to leave his bag with his companions in his absence, although they would accept leaving it on an empty coast. Can they cross the river?

2. Sasha has 5 bags with candies. Checking all possible pairs of bags, he has noticed that the total numbers of candies in these pairs assume only three values: 53, 66, and 79. How many candies are there in each bag?

3. A triangular piece of paper with sides a, b, c is folded so that the vertex opposite to side c touches this side. Assume that, in the obtained quadrilateral, the two angles at the fold line are equal. Determine the lengths of the parts into which the side c is divided by that vertex.

4. Given a square board 20×12 whose central square 2×2 has been cut out, is it possible to cut it into L-tetraminoes (i.e. figures consisting of four unit squares in the shape of letter L)?

5. If x and y are arbitrary nonnegative real numbers with the sum not exceeding 1, prove that $x^2 + 8x^2y^2 + y^2 \leqslant 1$.

6. Points K and L are taken on sides AB and BC respectively of an isosceles triangle ABC with $AB = BC$ so that $AK + CL = \frac{1}{2}AB$. Find the locus of the midpoint of segment KL.

7. Several distinct natural numbers are written in a line. We call a pair of neighboring numbers *bad* if their sum is divisible by 7 and the larger of the two numbers is on the left, or if their sum is *not* divisible by 7 and the larger number is on the right. Every minute, the two numbers in some bad pair change places. Prove that, at some moment, such changes must stop.

8. There are 1111 plates in a line, containing $1, 2, 3, \ldots, 555, 556, 555, 554, \ldots, 2, 1$ walnuts in this order. In a step, one can move several walnuts (at least one) to the neighboring plate on the left, or eat any number (at least one) from the leftmost plate. Petya and Vasya alternately perform steps, Petya playing first. A player who cannot make a step loses. Which player can win no matter how his opponent plays?

9. Let P be the product of some eight consecutive natural numbers, and Q the smallest perfect square with $Q > P$. Prove that the difference $Q - P$ is a perfect square.

10. In a regular heptagon $ABCDEFG$ of side 1, the diagonals AD and CG intersect at point H. Prove that $FH = \sqrt{2}$.