# Geometry

**1.** Let $ABC$ be a triangle with a right angle at $C$ and let $M$ be the midpoint of the side $AB$. Point $Q$ on side $CB$ is such that $\frac{BQ}{QC} = 2$. Prove that $\angle QAB = \angle QMC$.

**2.** The incircle of an isosceles triangle $ABC$ with $AB = BC$ is tangent to $BC$ and $AB$ at $E$ and $F$ respectively. A half-line trough $A$ inside the angle $EAB$ intersects the incircle at points $P$ and $Q$. The lines $EP$ and $EQ$ meet the line $AC$ at $P'$ and $Q'$. Prove that $P'A = Q'C$.

**3.** Let $P$ be an arbitrary point inside a tetrahedron $ABCD$. Denote by $R$ the circumradius of the tetrahedron and by $x$ the distance from $P$ to the circumcenter. Prove the inequalities

$$(R + x) \cdot (R - x)^3 \leqslant PA \cdot PB \cdot PC \cdot PD \leqslant (R + x)^3 \cdot (R - x).$$

**4.** Let $ABC$ be a scalene triangle. Denote by $I_A$ the center of the excircle at side $BC$ and by $A_1$ its tangency point with the corresponding side. Points $I_B$, $I_C$, $B_1$, $C_1$ are defined analogously. Show that the circumcircles of triangles $AI_AA_1$, $BI_BB_1$ and $CI_CC_1$ have two common points.