Combinatorics and Logic

1. Four students sit on a round table. The philologist sits opposite to Luzin and next to the historian. The mathematician sits next to Lebedev. Lihachev's neighbors are Solovyev and the physicist. What is Luzin's occupation?

2. Numbers $1, 2, 3, \ldots, 2012$ are written on the board. Petya erases them one by one. Show that he can do it in such a way that, in every moment, the sum of numbers on the board is composite.

3. On each side of a triangle, 10 points are taken. Each vertex of the triangle is joined by a segment to each of the points on the opposite side. In how many regions at most can these segments divide the triangle?

4. Pauli and Bohr alternate playing with a heap of molecules, Pauli playing first. Initially there are $99!$ molecules. In each move, a player may take not more than 1% of the molecules from the heap. The player who cannot perform a legal move loses. Who has a winning strategy?

5. A square building of size $40m \times 40m$ is divided by corridors into $5m \times 5m$ squares. At each vertex of each smaller square there is a switch. Flipping a switch simultaneously turns the light (on$\leftrightarrow$off) in all 5-meter corridors going out of that vertex. Initially, the building is in complete darkness, and a guard starts from one of its corners. He can only walk along lit corridors. A switch can be flipped arbitrarily many times. Can he move in such a way that, from some moment on, he can walk from any point to any other without flipping any more switches?

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