Algebra and Number Theory

1. Find the smallest natural number that gives pairwise different remainders when divided by 2, 3, 4, 5, 6, 7, 8, 9, 10.

2. The sequence $\{a_n\}$ is defined recursively: $a_1 = \frac{1}{2}$ and $a_n = \frac{a_{n-1}}{2n \cdot a_{n-1}+1}$ for $n > 1$. Evaluate the sum $a_1 + a_2 + \ldots + a_{2012}$.

3. Real numbers $a_1, a_2, \ldots, a_n$ satisfy $a_m + a_{m+1} + \ldots + a_n \geqslant m + (m+1) + \ldots + n$ for any natural number $m \leqslant n$. Prove that $a_1^2 + \ldots + a_n^2 \geqslant 1^2 + 2^2 + \ldots + n^2$.

4. Find all pairs of natural numbers (a, b) such that b divides a^2, a divides b^2, and $a+1$ divides $(b+1)^2$.

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