Combinatorics and Logic

1. The 77 inhabitants of an island, each a knight or a liar, stood in a circle. It is assumed that their weights are mutually distinct. On the question "Is one of your neighbors a liar lighter than you?" everyone answered "yes". After a break, they stood in a circle in a different order. Show that, on the question "Is one of your neighbors a knight lighter than you?", at least two persons answered "yes".

2. There are 101 saucers on a circle, each with a candy. Lillebror chooses a natural number $f < 101$, and then Karlsson, knowing Lillebror's number, chooses a natural number $s < 101$. Lillebror takes the candy from some saucer. Starting from this saucer, Karlsson counts the s-th saucer clockwise and takes the candy from it. Next, starting from that saucer, Lillebror takes the candy (if there is one) from the f-th saucer clockwise. Then, starting from that saucer, Karlsson takes the candy (if there is one) from the s-th saucer clockwise, etc. At most how many candies can Karlsson make sure to collect, no matter how Lillebror plays?

3. There were 10 teams on a football tournament and every two teams played one match, but the scoreboard was lost and only the numbers of goals scored and conceded for each team were saved. However, from these 20 numbers, the results of all matches can be reconstructed. At least how many of these 20 numbers can be zeroes?

4. Each vertex of a polyhedron with n vertices is assigned two positive numbers, blue and red, where the sum of blue numbers equals the sum of red ones. In one step, one can alter the blue numbers in the endpoints of an edge of the polyhedron, so that they remain positive and their sum is unchanged. Prove that in at most $n-1$ steps one can achieve that each blue number be equal to the corresponding red number.

5. Given natural numbers r and n, $r \leqslant n$, consider all possible n-tuples $(k_1, k_2, \ldots, k_n)$ of nonnegative integers such that $k_1 + k_2 + \cdots + k_n = r$ and $k_1 + 2k_2 + \cdots + nk_n = n$. Each such n-tuple is assigned the quotient $\frac{1}{k_1!k_2!\cdots k_n!}$ (where $0! = 1$). Prove that the sum of all these quotients equals $\frac{(n-1)!}{(n-r)!r!(r-1)!}$.

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